11.3

Walking the . . . Curve?

Domain, Range, Zeros, and Intercepts

LEARNING GOALS

In this lesson, you will:

- Describe the domain and range of quadratic functions.
- Determine the *x*-intercept(s) of a graph of a quadratic function.
- Understand the relationship of the zeros of a quadratic function and the *x*-intercepts of its graph.
- Analyze quadratic functions to determine intervals of increase and decrease.
- Solve a quadratic function graphically.

KEY TERMS

- vertical motion model
- zeros
- interval
- open interval
- closed interval
- half-closed interval
- half-open interval

The first liquid-fueled rocket in history was created by Robert Goddard, who was a university professor and inventor, claiming more than 200 patents.

Goddard led a team that launched more than two dozen rockets in a span of 15 years. He was ridiculed in the press for his research, and his efforts received little support.

Yet, his invention paved the way for spaceflight, over 40 years later!

PROBLEM 1 Model Rocket

Suppose you launch a model rocket from the ground. You can model the motion of the rocket using a *vertical motion model*. A **vertical motion model** is a quadratic equation that models the height of an object at a given time. The equation is of the form

$$g(t) = -16t^2 + v_0t + h_0,$$

where $g(t)$ represents the height of the object in feet, t represents the time in seconds that the object has been moving, v_0 represents the initial vertical velocity (speed) of the object in feet per second, and h_0 represents the initial height of the object in feet.

1. Why do you think it makes sense that this situation is modeled by a quadratic function?

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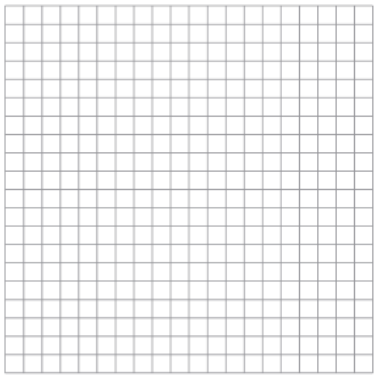
Suppose the model rocket has an initial velocity of 160 feet per second.

2. Write a function, $g(t)$, to describe the height of the model rocket in terms of time t.

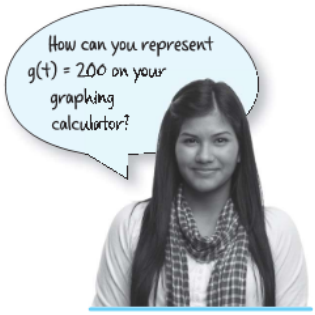
3. Describe the independent and dependent quantities.

4. Use a graphing calculator to graph the function. Sketch the graph and label the axes.



5. Use a graphing calculator to answer each question.

a. What is the height of the model rocket at 6 seconds?



b. After approximately how many seconds is the model rocket at a height of 200 feet?

c. What is the maximum height of the model rocket? When is the rocket at its maximum height?

6. You can use a graphing calculator and intersection points to determine the x-intercepts of a quadratic function.

a. What linear function can you graph along with the quadratic function to determine the x-intercepts? Explain your reasoning.

b. Determine the x-intercepts of $g(t)$. Then, interpret the meaning in terms of this problem situation.

The x-intercepts of a graph of a quadratic function are also called the **zeros** of the quadratic function.

You can use a graphing calculator to determine the zeros of a quadratic function.

Step 1: Press **2ND** and then **CALC**. Select **2: zero**.

Step 2: Determine the left and right bounds for each point that appears to be a zero. Then press **ENTER**.

7. Identify and describe the domain of the function in terms of the:

a. mathematical function you graphed.

b. contextual situation.

8. Identify and describe the range of the function in terms of the:

a. mathematical function you graphed.

b. contextual situation.

9. How is the range of a quadratic function related to its absolute maximum or minimum?

An **interval** is defined as the set of real numbers between two given numbers. To describe an interval, use this notation:

- An **open interval** (a, b) describes the set of all numbers between a and b, but not including a or b.
- A **closed interval** $[a, b]$ describes the set of all numbers between a and b, including a and b.
- A **half-closed** or **half-open interval** $(a, b]$ describes the set of all numbers between a and b, including b but not including a. Or, $[a, b)$ describes the set of all numbers between a and b, including a but not including b.

Intervals that are unbounded are written using the symbol for infinity, ∞.

The interval $[a, \infty)$ means all numbers greater than or equal to a.
The interval (a, ∞) means all numbers greater than a.

10. Use interval notation to describe the interval in which:

a. all numbers are less than a.

b. all numbers are less than or equal to a.

c. a is any real number.

11. Use interval notation to describe the interval of the domain in which the model rocket function is:

a. increasing.

b. decreasing.

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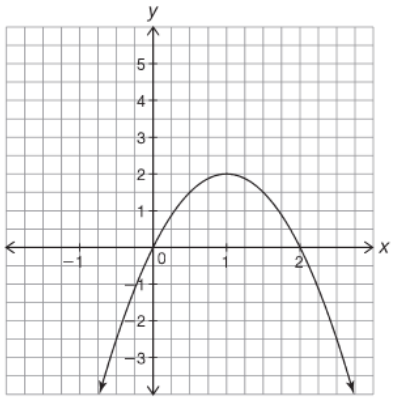
12. How does the absolute maximum or absolute minimum help you determine each interval?

PROBLEM 2 Intervals of Increase and Decrease

For each function shown, identify the domain, range, zeros, and the intervals of increase and decrease.

1. The graph shown represents the function $f(x) = -2x^2 + 4x$.



Domain:

Range:

y-intercept:

Zeros:

Interval of increase:

Interval of decrease:

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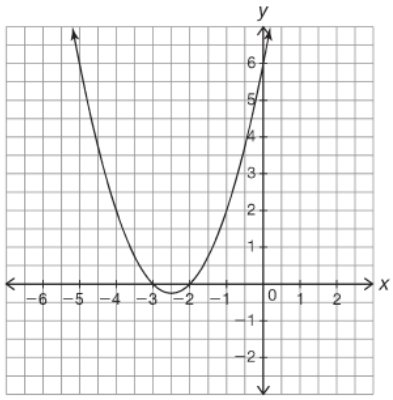
2. The graph shown represents the function $f(x) = x^2 + 5x + 6$.



Domain:

Range:

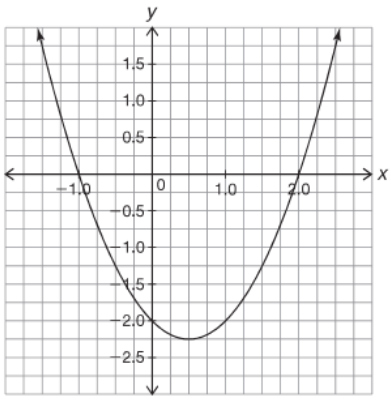
y-intercept:

Zeros:

Interval of decrease:

Interval of increase:

3. The graph shown represents the function $f(x) = x^2 - x - 2$.



Domain:

Range:

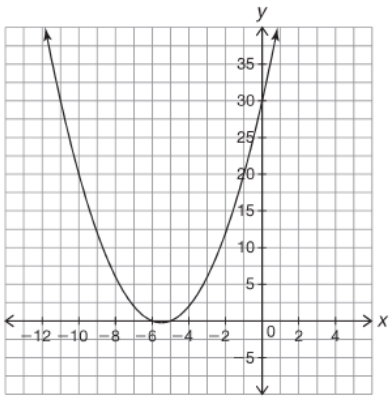
y-intercept:

Zeros:

Interval of decrease:

Interval of increase:

4. The graph shown represents the function $f(x) = x^2 + 11x + 30$.



Domain:

Range:

y-intercept:

Zeros:

Interval of decrease:

Interval of increase:

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Be prepared to share your solutions and methods.